PERCOLATION ANALYSIS OF A WIENER RECONSTRUCTION OF THE IRAS 1.2 Jy REDSHIFT CATALOG

Capp Yess and Sergei F. Shandarin

Department of Physics and Astronomy

University of Kansas, Lawrence, KS 66045

and

Karl B. Fisher

Institute for Advanced Studies

Olden Lane, Natural Sciences, Bldg E,

Princeton, NJ 08540

Received ____________________; accepted ____________________

ABSTRACT

We present percolation analyses of Wiener Reconstructions of the IRAS 1.2 Jy Redshift Survey. There are ten reconstructions of galaxy density fields in real space spanning the range $\beta = 0.1$ to 1.0, where $\beta = \Omega^{0.6}/b$, Ω is the present dimensionless density and b is the bias factor. Our method uses the growth of the largest cluster statistic to characterize the topology of a density field, where Gaussian randomized versions of the reconstructions are used as standards for analysis. For the reconstruction volume of radius, $R \approx 100h^{-1}$ Mpc, percolation analysis reveals a slight 'meatball' topology for the real space, galaxy distribution of the IRAS survey.

Subject headings: cosmology-galaxies:clustering-methods:numerical

1. Introduction

Quantifying the distribution of galaxies in the visible universe has been one of the primary objectives of the study of the Large Scale Structure of the universe for several decades. With the compilation of early two dimensional galaxy catalogs (for example, Zwicky *et al.* (1961) and Shane & Wirtanen (1967)), astronomers noted structures indicative of clustering and evolution. More recent redshift surveys (see, Davis *et al.* (1982); de Lapparent, Geller & Huchra (1986); Giovanelli & Hayes (1985); Tully & Fisher (1987); Lawrence *et al.* (1996)) produced three-dimensional galaxy distributions and revealed structures such as voids (for example, Kirshner *et al.* (1981)), and filaments and sheets of galaxies (Geller & Huchra 1989). Angular and spatial correlation functions (Peebles 1980 and references therein) were employed as initial attempts to distinguish the galaxy surveys from random distributions and from comparisons with theoretical models.

In 1982, Zel'dovich proposed that the large scale structure of the universe could be characterized by its topology, and a multitude of statistical measures have been developed since then to quantify the topology of a distribution: the percolation threshold (Shandarin 1983; Shandarin & Zel'dovich 1983); the genus (Gott, Melott & Dickinson 1986); the contour crossing (Ryden 1988); random walk statistics (Baugh 1993); Minkowski functionals (Mecke, Buchert & Wagner 1994); and minimal spanning tree characteristics (Bhavsar & Splinter 1996). Many obstacles have been overcome in the refinement of the measures above including boundary and selection effects, discreteness, local biasing, and error estimation. For example, recent topological analyses of the CfA redshift survey by percolation (de Lapparent, Geller & Huchra 1991) and genus (Vogeley *et al.* 1994) report important and complementary findings. The topology and power spectrum of the IRAS survey has been examined in the context of the Queen Mary and Westfield College, Durham, Oxford and Toronto (QDOT) survey by Moore *et al.* (1992). The results of every report cited above are consistent with the findings of Gott *et al.* (1989): On scales significantly larger than

the correlation length the topology is sponge-like. A sponge topology is characterized by equivalent, over- and under-dense, multiply connected regions both of which percolate and are completely interlocked. These findings are consistent with the scenario that large scale structure developed from initial random Gaussian density fluctuations present at the epoch of recombination. In addition, for smoothing lengths comparable to or smaller than the correlation length slight shifts towards a 'meatball' (Gott *et al.* 1989; Moore *et al.* 1992) and 'bubble' (Vogeley *et al.* 1994) topology have been reported. The scope of the upcoming Sloan Digital Sky Survey (Gunn & Weinberg 1995) promises even more significant results.

We use a percolation analysis that tracks two parameters in order to measure the connectivity of a Wiener reconstruction of the IRAS 1.2 Jy Redshift Survey and to estimate the spectral index of its associated power spectrum. The Wiener filter (WF) takes a three dimensional, redshift map with the excluded zones filled by interpolation and converts it to a noise-free, full sky, real space map. Our percolation code computes the normalized volume of the largest structure as a function of the filling factor for a topological comparison. The number of clusters statistic has been shown to be sensitive to the index of the power spectrum for a simple power law relationship (Yess & Shandarin 1996). The method has been used for studying the properties of voids (Sahni, Sathyaprakash, & Shandarin 1994) and the geometry of mass clumps (Sathyaprakash, Sahni, & Shandarin 1995) in cosmological N-body simulations. The largest structure can be an over-dense region (cluster [1]) or an under-dense region (void). By comparing the growth of the largest structures (as functions of the filling factor) in a distribution with their growth in a randomized version of the

[1]The terms cluster and void in this context refer to high and low density regions respectively and not to the common astronomical meanings. Also, for the IRAS survey the terms under-dense and over-dense refer to galaxy densities; whereas, for N-body simulations the terms refer to mass densities.

distribution the distribution topology can be characterized. A Gaussian randomized version of a distribution is by definition a structureless field with the same power spectrum as its parent, so that any distribution that percolates at a lower filling factor than its randomized version is considered more connected than a random field, and hence, a connected network; whereas, any field that percolates at a higher filling factor than its randomized version is considered isolated or clumpy. In addition, reconstructions of N-body simulations (with and without Wiener filters) over the spectral range $n = -2, -1$ and 0 are analyzed for comparison.

In §2 we describe the the density fields derived from Wiener reconstructions of the IRAS 1.2 Jy Redshift Survey (Fisher *et al.* 1995a) and randomized versions which preserve the underlying power spectrum but are Gaussian fields. In §3 we detail the percolation method and parameters we have developed for analysis and comparison. Our results are presented and scrutinized in §4 with conclusions to follow in §5.

2. The Density Fields

The core of this study are Wiener reconstructions of the *real* space density field formed from the IRAS 1.2 Jy Survey (Fisher *et al.* 1995b). This survey is an extension of the 1.936 Jy flux limited survey of Strauss *et al.* (1992) compiled from the Infrared Astronomical Satellite Point Source Catalog (1988; PSC). The survey contains 5321 galaxies which cover 87.6 per cent of the sky. The 12.4 per cent of the sky that is missing from the survey is the Zone of Avoidance ($\pm 5^{\circ}$ from the galactic plane) and a few confused regions or areas lacking coverage. For specifics of the galaxy selection criteria, the participating telescopes, data reduction techniques and results, the derived selection function, and the galaxy distribution see Fisher *et al.* (1995b).

2.1. Wiener Reconstructions

A Wiener reconstruction method is used to convert an interpolated redshift density field to a real space density field while suppressing noise. This approach is valid in the context of linear theory implying significant smoothing which is an aspect of the filter. The Wiener filter reconstruction method has been employed in many fields (Rybicki & Press 1992) to enhance a signal in the presence of noise. In cosmology, one use of the method is the minimum variance reconstruction of the real space density function from an incomplete and sparsely sampled galaxy distribution in redshift space. The process depends upon an expectation of the clustering properties of the real underlying density field being probed by the galaxy survey. In the cosmological case, the underlying density field is assumed to be Gaussian up to near the stage of non-linearity.

The WF algorithm as it applies to the IRAS 1.2 Jy galaxy survey is discussed in detail in Fisher *et al.* 1995a; for completeness we give a brief summary here. The filtering is applied to a three dimensional decomposition of the redshift space galaxy density field in an orthogonal basis set of the spherical harmonics and spherical Bessel functions. The decomposition is truncated with l ranging from $0 \leq l \leq 15$ (with $-l \leq m \leq l$) and $0 \leq k_n r \leq 100$ as a compromise between resolution and the number of expansion coefficients. The WF reconstruction technique depends on the assumed linear theory growth parameter $\beta = \Omega^{0.6}/b$ where Ω is the the current cosmic density and b is the linear bias parameter. We investigate a set of ten reconstructions spanning the range $\beta = 0.1$ to $\beta = 1.0$. In each case, the real space density field is reconstructed on a 64^3 grid with sides of length $200h^{-1}$ Mpc (20,000 km/s).

To compute the Wiener reconstructions the first step is to compute the redshift space harmonics in the spherical harmonics and spherical Bessel functions basis. This is analogous to computing the Fourier components in the analysis of the power spectrum.

The redshift harmonics are distorted from the values that would be measured in a perfect real space galaxy distribution. First, the actual galaxy distribution is sparsely sampled and this results in a statistical uncertainty or shot noise in the estimated harmonics. Second, peculiar velocities introduce a systematic distortion due to the coherent infall and outflow around over-dense and under-dense regions. In linear theory, this redshift distortion can be computed if the value of β is known; the spherical basis function is convenient here since the distortion is in the form of a matrix which couples the radial modes of the expansion.

In the absence of shot noise, the real space harmonics could be recovered by a direct inversion of the coupling matrix. Shot noise makes this inversion highly unstable. The Wiener filter is a smoothing algorithm which is designed to make the inversion in the presence of noise optimal in the sense of minimum variance. It depends on the 'prior' which is the knowledge of the clustering of the underlying field. Essentially, the Wiener filter is the ratio of the variance in the signal (determined from the assumed prior power spectrum) to the sum of the variance in both the signal and noise (determined by the amplitude of the shot noise).

The result is a density field in real space centered on the local group. Two important properties of the reconstruction method are an effective smoothing of the resultant field which increases with radius due to the limited resolution caused by truncating the harmonic expansion at $l_{max} = 15$, and the increased attenuation of the signal as a function of radius because the Wiener filter is dependent on the shot noise. The direct implication of truncating the harmonic expansion at l_{max} for fields sampled on a 64^3 grid is that at distances of $R = 30$ mesh units ($\approx 100h^{-1}$Mpc) the minimum resolution is approximately $R \times (\pi/l_{max}) \approx 6$ mu The effects of smoothing will be examined in detail in section §4. For detailed explanations of the Wiener filter reconstruction method for different response functions see Fisher *et el.* (1995a) and Zaroubi *et al.* (1995).

In addition to the reconstructions of the IRAS data, we produced reconstructions of

cubic (L^3) density fields derived from N-body simulations with power law initial spectrum ($P(k) = Ak^n$) for $n = -2, -1$, and 0 evolved to the stage where scales of the size L/4 were approaching nonlinearity (Melott & Shandarin 1993). Assuming that the rms fluctuation in the number of galaxies is approximately equal to the rms mass fluctuation, both are unity within spheres of radius $8h^{-1}$ Mpc. So, by identifying the stage where L/4 becomes nonlinear with the present, a rough estimate of the size of a mesh cell is $3.1h^{-1}$ Mpc. These reconstructions were produced with and without Wiener filtering in order to systematically study the effects of harmonic expansion and Wiener filtering. In addition, the N-body simulations are reconstructed with the IRAS WF and not a WF based on the clustering and noise in the simulations. Using all the particles from the simulations to compute their harmonics and smoothing with a WF which corresponds to the sampling density of the IRAS 1.2 Jy survey assures that the N-body reconstructions show the same resolution as the IRAS reconstructions. The volume of the N-body reconstructions was chosen to match the local mass density of the IRAS galaxy distribution at 500 km/s ($5h^{-1}$ Mpc), approximately $0.045/h^{-3}$ Mpc^3 to allow for visual comparison.

2.2. Gaussian Randomizations

We produce Gaussian random fields by two methods in this study. Reconstructions are expanded in a spherical harmonic basis set and are also randomized in this basis; whereas, original N-body simulation density fields which are not reconstructed are Fourier transformed and then randomized in k-space. The crux of both processes is the randomization of the phases while retaining the original power spectrum of the parent field. For reconstructions (with and without Wiener filtering) this is accomplished by multiplying the density, ρ_{lmn}, for $m > 0$ by $e^{i\phi}$, where ϕ is a random variable in the range $0 \leq \phi \leq 2\pi$. The $m = 0$ term is multiplied by $\sqrt{2}\cos\phi$, and the $m < 0$ terms are determined using a

reality condition for ρ. The randomized versions of fields derived from N-body simulations are created by multiplying the components of all k-space vectors by $\cos\phi$, and a reality condition again assigns values to coefficients in the lower half of k-space.

3. Percolation Method

The percolation methodology [2] we employ analyzes galaxy (IRAS) or mass (N-body) distributions as well as void distributions. The intent is to characterize the topology of both distributions, and to estimate the slope of the power spectrum of the density field. The discriminator between mass sites and voids is the density threshold, and it is smoothly varied to establish contours separating clusters and voids. Void and galaxy percolation are analogous to mass percolation so for simplicity percolation will be discussed in the context of mass percolation except where distinction is needed. As the density threshold is varied three parameters are tracked: the filling factor, the volume of the largest structure (for both over-dense and under-dense structures), and the number of isolated structures.

The filling factor is the fractional volume of all mass sites identified in the distribution for a given density threshold. It is equivalent to the cumulative distribution function for the clusters and the volume fraction of Gott *et al.* (1989) for Gaussian distributions. For clusters, the filling factor grows from a minimum value of zero to a maximum of one as the density threshold is systematically lowered. The filling factor serves as the independent variable for our functions to allow for a fair comparison between different density fields.

The second parameter, the volume of the largest cluster, is a stable indicator of the percolation transition and is used to assess the topology of the field. The volume is reported

---

[2] For a detailed description and evaluation of the percolation technique used in this study see Yess & Shandarin (1996).

in units of the filling factor (the ratio of the largest cluster to the total volume of all clusters) as a function of the filling factor, and a rapid increase in the volume indicates the filling factor associated with the percolation transition. This transition represents a change from a clumpy to a connected topology for the field. For clusters it is a change from a meatball to a sponge topology, and for voids it is a change from a bubble to a sponge topology. Gaussian fields are used as standards of comparison to characterize the topology of density fields. A field which percolates at a smaller filling factor than its Gaussian counterpart displays a shift towards a connected topology, while a field that percolates at higher filling factors displays a shift towards a clumpy topology.

The number of clusters statistic is sensitive to the slope of the power spectrum of a field (Yess & Shandarin 1996). This implies that for a field described by a power spectrum of the form $P(k) = Ak^n$ that the number of clusters statistic is sensitive to the spectral index, n. In fact, the maximum of the statistic is a function of n, and can be used to estimate the effective spectral index of a mass distribution.

4. Results

Topological analysis of modern redshift surveys has focused on the two aspects of the galaxy distributions mentioned above: a quantitative assessment of the connectedness of the structure, and the slope of the power spectrum. In addition to the difficulties of assessing boundary effects and error estimations, a major obstacle for all current methods employed to describe the spatial distribution of galaxies is the lack of resolution resulting from the sparse sampling achieved in existing surveys.

The resolution of any representation of a galaxy survey is ultimately a function of the mean galaxy density of the survey and the chosen smoothing method. In this respect

the IRAS 1.2 Jy survey presents good prospects with the average galaxy number density higher than the QDOT survey value in the region $R \approx 100h^{-1}$ Mpc. However, different groups have utilized different smoothing routines to produce density fields from the galaxy distributions of the surveys. For example, the smoothing methods of Moore *et al.* (1992) in their analysis of the QDOT survey are typical, but differ significantly from those utilized in the spherical harmonic reconstruction of our data. Moore and collaborators used a constant Gaussian filtering width determined by the inter-galactic spacing at the edge of the QDOT survey, $\lambda = [S(r_{max})]^{-1/3}$, where $S(r)$ is the radial selection function. In a magnitude limited sample this choice ensures that the density field is not under-sampled while providing an unprecedented number of resolution elements for the QDOT survey. In contrast, the spherical harmonic reconstruction of the IRAS 1.2 Jy survey implies a variable smoothing with radius due to the finite cutoff of l in the spherical harmonic expansion. In addition, the Wiener filter suppressed the amplitude of the field as a function radius to mitigate the effects of increasing shot noise (as determined by the selection function). The effect of the variable smoothing in the density field is evident in the results reported below.

Like all statistical measures our parameters are sensitive to the resolution and number density of the data, but they are relatively robust with respect to boundary effects and scale. Since the volume of the largest structure is normalized to the filling factor, and the number of clusters statistic can be normalized to the volume of the survey, the geometry and size of the survey does not determine their analytic behavior. We exploit the stability of our parameters by analyzing spherical subregions of the survey in order to examine the effects of variable smoothing and any local bias against a fair sample. A measure of the stability of our parameters are the errors presented for the results from N-body simulations with multiple realizations. In all instances the error bars represent 1σ deviations over four realizations. We do not estimate errors in the results for the IRAS reconstructions, but rather rely on trends in the versions varying in β, over the range $0.1 \leq \beta \leq 1.0$, to

determine conclusions.

4.1. Largest Structures

Largest structure results for all versions of the Wiener Reconstructions are shown in Figure 1 for both clusters and voids. The top panels show the growth of the largest structures for a field sampled on a cubic grid, 64 mu ($200h^{-1}$ Mpc) to a side [3]. If we consider the percolation threshold to be the first significant jump in the value of the largest structure statistic (de Lapparent, Geller & Huchra 1986) then percolation happens for all versions in the range $0.024 \leq ff \leq 0.05$ for clusters and between $0.01 \leq ff \leq 0.022$ for voids. For cluster percolation, fields with larger values of β percolate at smaller filling factor values, while for voids the trend is generally reversed. This means the larger the average cosmic density or the smaller the bias factor the more connected the clusters tend to be. The fact that the percolation threshold is the distinguishing difference between the curves demonstrates the sensitivity of this parameter as suggested by Shandarin (1983). Alternately, the high sensitivity may also cause problems in noisy samples as reported by Dekel & West (1985). However, rigorously determining a percolation threshold value is not important to the analysis in this study and is used here only for illustration. In this study the shape of the largest structure function over its entire range will be used as a comparison to characterize the topology of a field.

Another important feature of the largest structure statistic is the high initial values at low filling factors for all versions of the Wiener Reconstructions. This indicates that the largest structure is always dominant which indicates a problem with the size of the

---

[3] The result of a spherical harmonic reconstruction is a spherical field, and the (64^3 m.u.) density field analyzed is the largest cubic subregion of the original output.

sample. In a statistically fair sample, there would be a multitude of small clusters at the high density cutoffs beginning the percolation process, and the largest cluster would emerge from the field as the percolation process caused clusters to join together. The fact that the statistic has a non-vanishing initial value is indicative of the relatively small sample size of this survey for the purpose of this statistic and the level of smoothing introduced by the reconstruction process. For comparison, see the percolation results of N-body simulations in Figure 3 and reference Yess & Shandarin (1996).

A lack of resolution at large distances explains the relationship between the results of the upper and lower panels of Figure 1. The growth of the largest structure function is virtually identical for the two cases except for a near doubling in the filling factor. This means that all information about structure is contained in the reduced spherical region bounded by $R \approx 100h^{-1}$ Mpc (30 mu), and that the excess volume in the cube does not affect the structures but only contributes to a reduction in the filling factor. The implication is that the value of the field outside $R \approx 100h^{-1}$ Mpc is featureless and roughly equal to the mean density. This is because attenuation of the signal is a function of radius due to the effective smoothing of the spherical harmonic reconstruction and the loss of detail after Wiener filtering. The effects of these two operations will be examined separately below.

Restricting the analysis to an even smaller volume ($R \approx 30h^{-1}$ Mpc (10 mu), not shown), reveals similar percolation curves to those of larger volumes for the growth of the largest clusters except that volume effects for clusters are exaggerated to the point where the largest cluster is associated with the highest density peak and its volume is never less than half the volume of all clusters combined. For voids the curves are also similar at the outset, but rise much slower so $ff \approx 0.3$ when the volume of the largest void approaches unity.

In Figure 2, we display the results of a systematic study of the effects of spherical harmonic reconstruction and Wiener filtering separately on N-body simulations. The results

of analysis of four realizations of N-body simulations characterized by an initial power law power spectrum of the form $P(k) \propto k^{-1}$, evolved to the stage where $\lambda = \lambda_f/8$ (where λ_f is the fundamental wavelength) is approaching non-linearity [4]. The upper panels show the results of percolation analysis for the simulations and demonstrate that the topology of the structure is similar throughout the volume and not a function of radius. The difference between percolation in a cube and a sphere is also insignificant. It is also important to note that the rapid growth of the largest structure for random Gaussian fields (light lines) starts at a filling factor of $ff \approx 0.16$ for both clusters and voids, which is the expected value. The interpretation of this data is that the topology of these simulations is characterized by a very connected cluster network and slightly more isolated voids compared to Gaussian fields.

The middle panels show the effect of spherical harmonic reconstruction alone and in conjunction with Wiener filtering. The reconstruction process eliminates the distinction between the topologies of clusters, voids and the random fields in a spherical volume of radius, $R \approx 100h^{-1}$ Mpc, and introduces some distortion in the curves at low filling factors. In addition, the Wiener filter removes most of the small scale structure demonstrated by the almost immediate percolation of the fields and the reduction of error bars. It is easy to understand this effect of the Wiener filter because it reduces the range of the density values in the reconstructed density field by one-fourth. In order to regain a measure similar to that of the original density fields, the effects of the reconstruction and filtering have to be minimized by restricting the analysis to smaller radii. The smoothing effect of the reconstruction is less and the signal to noise ratio is better for smaller radii, so that the topology of the original field can be recognized in a volume limited sample at $R \approx 30h^{-1}$ Mpc if the reconstruction alone is applied to the simulation (bottom left panel). The

[4] For a detailed discussion of the N-body simulations used in this study see Melott & Shandarin (1993).

Wiener filtering still distorts the original topology even in this restricted volume as shown in the bottom right panel. An important feature for the interpretation of percolation results is that the largest structures in the random Gaussian realizations behave, making allowance for survey volume effects and smoothing, generally as expected in all cases except the middle right panel.

Finally, Figure 3 shows the largest structure statistics for Wiener reconstructions of the IRAS 1.2 Jy Survey for various β values. Each version displays a similar result with voids percolating at lower filling factors than the Gaussian counterparts, and clusters percolating similarly to the Gaussian fields. These results imply a well connected void distribution with a generally sponge like or slightly meatball cluster distribution. Volume and smoothing effects are again evident in each field demonstrated by the high values of the largest structure statistic at low filling factor values. This problem is more apparent in the cluster analysis of the original IRAS reconstructions than for voids or clusters in randomized IRAS reconstructions. Although the lack of resolution prevents a strong characterization of the topologies represented in the data, the results are consistent with the slight meatball topology shift reported by Moore *et al.* (1992) and Gott *et al.* (1989) for similar local volumes. Results for volumes with $R \approx 30h^{-1}$ Mpc (not shown) are inconclusive.

4.2. Number of Clusters

It has been established that the maximum of the number of clusters statistic reflects the slope of the power spectrum for density fields described by a simple power law (Yess & Shandarin 1996). This is true for Gaussian fields and randomized density fields derived from N-body simulations over a wide range of evolutionary stages. This statistic can easily be normalized by the volume of the field so that different samples can be directly compared; however, the values of the maximum are small enough in this study that the raw data is

presented for clarity. The results of percolation analysis of clusters and voids are presented in Figure 4 for both reconstructed IRAS survey ($\beta = 0.1, 0.5, 1.0$) and N-body simulation ($n = 0, -1, -2$) fields. The most notable feature of the data is that the maxima for N-body simulation fields are two orders of magnitude below those of the original fields before they were reconstructed. This reduction in the number of clusters (voids) is a direct result of the smoothing and attenuation of the Wiener Reconstruction procedure. The effect is to reduce the signal below the level where distinctions can be made between fields characterized by different spectra. Another indication that the resolution of the reconstructions is insufficient for percolation analysis are the many local maxima in the statistic.

5. Conclusions

The Wiener reconstruction technique has proven successful in reconstructing the angular density fields of galaxies (Lahav *et al.* 1994); the temperature fluctuations of the Cosmic Microwave Background (Bunn *et al.* 1994); real space density, velocity and gravitational potential fields (Fisher *et al.* 1995a); and predicted full sky density fields (Zaroubi *et al.* 1995). In this study we apply percolation analysis to full sky Wiener Reconstructions of the IRAS 1.2 Jy Redshift Survey. We find that our results are consistent with the conclusions of other studies that report a small shift towards a meatball topology for the survey region, however we would like to stress that our analysis was in real space.

The Wiener reconstruction technique smoothes the density field based on the inter-galaxy separation as a function of radius and attenuates the high frequency components or small scale components resulting from shot noise. This results in a loss of resolution with distance which presents a challenge for percolation analysis. The largest structure statistic is robust enough to give an indication of the topology of the field under these conditions; however, the number of clusters statistic suffers too much from the loss of resolution to

give a measure of the slope of the associated power spectrum. Alternately, the number of clusters statistic has the potential to be developed into an indicator of whether or not the structures of a given field represent a fair sample for statistical purposes.

The prospects of analyzing a full sky density reconstruction in order to assess the topology of the large scale structure and associate that structure with initial fluctuations in the matter density field at the time of recombination are attractive. This study offers an optimistic picture that as more galaxies are added to surveys the statistical measures presented will produce accurate and convincing results. Until galaxy survey counts are increased enough to overcome the problems identified in this study, percolation can still be applied to point-wise galaxy distributions, density fields of highly sampled portions of the sky or ideally volume limited subsamples.

S. Shandarin acknowledges support from NASA Grant NAGW-3832, NSF Grant AST-9021414 and the University of Kansas Grant GRF96. We would like to thank David Weinberg for valuable suggestions and comments.

REFERENCES

Baugh, C. 1993, MNRAS, 264, 87

Bhavsar, S., & Splinter R., (1996) in progress

Bunn, E., Fisher, K. B., Hoffman, Y., Lahav, O., Silk, J., & Zaroubi, S. 1994, ApJ Lett., 432, L75

Davis, M., Huchra, J., Latham, D. W. & Tonry, J. 1982. ApJ, 253, 423

Dekel, A., & West, M. J. 1985, ApJ, 288, 411

Fisher, K.B., Lahav, O., Hoffman, Y., Lynden-Bell, D. & Zaroubi, S. 1995a MNRAS, 272, 885

Fisher, K. B., Huchra J. P., Strauss, M. A., Davis M., Yahil, A., & Schlegel, D. 1995b, ApJ Suppl., 100, 69

Geller, M. J. & Huchra, J. 1989, *Science*, 246, 897.

Giovanelli, R., & Haynes, M. P. 1985, AJ, 90, 2445

Gott, J. R., Melott, A. L., & Dickinson, M. 1986, ApJ, 306, 341

Gott, J. R., Miller, J., Thuan, T. X., Schneider, S. E., Weinberg, D. H., Gammie, C., Polk, K., Vogeley, M., Jeffery, S., Bhavsar, S., Melott, A. L., Giovanelli, R., Haynes, M. P., Tully, R. & Hamiltion, A. 1989, ApJ, 340, 625

Gunn, J. E. & Weinberg, D. H. 1995, *Wide-Field Spectroscopy and the Distant Universe, eds.* S. J. Maddox and A. *Aragón – Salamanca*, (Singapore : World Scientific).

Kirshner, R. P., Oemler, A., Schechter, P. L., & Shectman, S. A. 1981, ApJ, 248, L57

Lahav, O., Fisher, K. B., Hoffman, Y., Schaarf, C. A., & Zaroubi, S. 1994 ApJ Lett., 423, L93

de Lapparent, V., Geller, M., & Huchra, J. P. 1986, ApJ, 301, L1

de Lapparent, V., Geller, M., & Huchra, J. P. 1991, ApJ, 369, 273

Lawrence, A., Rowan-Robinson, M., Crawford, J., Saunders, W., Ellis, R. S., Frenk, C., Parry, I., Efstathiou, G., & Kaiser, N. 1995 in preparation

Mecke, K., Buchert, T. & Wagner, H. 1994, A&A, 288, 697

Melott, A. L., & Shandarin, S. F. 1993, ApJ, 410, 469

Moore, B., Frenk, C. S., Weinberg, D. H., Saunders, W., Lawrence, A., Ellis, R. S., Kaiser, N., Efstathiou, G., & Rowan-Robinson, M. 1992, MNRAS, 256, 477

Peebles, P. J. E., 1980. *The Large Scale Structure of the Universe*, Princeton University Press, Princeton, NJ.

Ryden, B. S. 1988, ApJ, 333, L41

Rybicki, G. B., & Press, W. H. 1992, ApJ, 398, 169

Sahni, V., Sathyaprakash, B. S., & Shandarin, S. F. 1994, ApJ, 431, 20

Sathyaprakash, B. S., Sahni, V., & Shandarin, S. F. 1996, ApJ Lett., 462, L5

Shandarin, S. F. 1983, Soviet Astron. Lett., 9, 104

Shandarin, S. F., & Zel'dovich, Ya. B. 1983, Comments Astrophys., 10, 33

Shane, C. D. & Wirtanen, C. A., 1967. *Publ.Lick Obs.,* 22, 1.

Strauss, M. A., Huchra, J. P., Davis, M., Yahil, A., Fisher, K., & Tonry, J. 1992a, ApJ Suppl., 83, 29

Tully, R. B., & Fisher, J. R. 1987, *Nearby Galaxies Atlas* (Cambridge: Cambridge University Press).

Vogeley, M. S., Park, C., Geller, M. J., Huchra, J. P., & Gott, J.R. 1994, ApJ, 420, 525

Yess, C., & Shandarin S. F. 1996, ApJ, 465, July1 issue

Zaroubi, S., Hoffmean, Y., Fisher, K. B., & Lahav, O. 1995, ApJ, 449, 446

Zel'dovich, Ya. B. 1982, Soviet Astron. Lett., 8, 102

Zwicky, F., Herzog, E., Wild, P., Karpowicz, M. & Kowai, C. T., 1961-1968. *Catalogue of Galaxies and Clusters of Galaxies*, California Institute of Technology, Pasadena

---

This manuscript was prepared with the AAS LaTeX macros v3.0.